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LIFTING REPRESENTATIONS OF KNOT GROUPS

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Abstract

Given a representation of a classical knot group onto a quotient group E/A, we address the classification of lifts of that representation onto E. The classification is given first in terms of classical obstruction theory and then, in many cases, interpreted in terms of the homology of covers of the knot complement. Applications include the study of dihedral, metacyclic, and metabelian representations. Properties of the restrictions of lifts to the peripheral subgroup are also studied.

Keywords: knot theory, knot groups.

1. Introduction

The topic of lifting knot group representations concerns the following problem: if a knot group, $\pi = \pi_1(S^3 - K)$, represents onto a quotient group, $G = E/A$, classify the liftings of that representation to E. For instance, Perko [1] proved that any representation of a knot group onto the symmetric group S_3 can be lifted to a representation onto S_4. (S_3 is the quotient of S_4 by the subgroup isomorphic to $Z_2 \times Z_2$ generated by $\{(12)(34), (13)(24)\}$.) A more basic result states that representations of a knot group to the dihedral group, D_{2p}, correspond to $H_1(M_2, Z_p)$, where M_2 is the 2-fold branched cover of S^3 branched over K. (Any such representation lifts the natural map of the knot group onto Z_2, a quotient of D_{2p}.) Much more comprehensive results have been obtained by Hartley [2, 3]; related work includes [4, 5, 6, 7, 8].

Our purpose here is to discuss this topic from the viewpoint of classical obstruction theory. That such an interpretation must exist is evident; the ultimate simplicity is more surprising. The methods apply most readily in the case that A is abelian and the quotient map splits over G; that is, E is a split extension of G

by an abelian group A. We will concentrate on this setting. In Section 2 we review the general obstruction theory that is needed. Section 3 describes a specific class of extensions, $A \to E \to G$, from which a number of interesting examples follow. For these extensions the obstructions that arise can be interpreted as classes in the cohomology of covering spaces of the knot complement; the connection to the cover is made via a basic result in cohomological algebra, Shapiro's Lemma. In Section 4 we indicate how a number of past results readily follow.

One aspect of the lifting problem that has received attention concerns the behavior of the liftings on the peripheral subgroup of the knot group. Results concerning the peripheral properties of the liftings have been obtained by Perko [1], Rickard [9], and Thistlethwaite [10], among others. Problems in this area are also easily placed in the context of the obstruction theory. Section 5 offers the first published proofs of these results and also illustrates new results that follow readily.

Results concerning the lifting problem have been presented in the context of knots in S^3, though many of the techniques were not completely restricted to this setting. The work presented here applies more generally. Initially we work with CW complexes, the role of the manifold appears later. Hence, the approach is quickly adopted to problems in link theory, general 3-manifolds, and higher dimensional questions. Section 6 describes a few such possible extensions.

Some of the previous work on lifting representations, especially that of Hartley [3], is clearly related to the obstruction theory presented here; Hartley's approach was to construct relative cycles in covering spaces and to use these to explicitly construct representations. Via duality, and some algebraic calculations, these relative classes correspond to the cohomological obstructions described here and the explicit representations Hartley described are the same as those which obstruction theory offers indirectly.

2. Review of Obstruction Theory

Let E be a split extension of a group G by an abelian group A; $A \to E \to G = E/A$. Fix a splitting, s; we can view G as a subgroup of E via s and A inherits the structure of a G-module via conjugation. Observe that the extension E is determined completely by the G-module A. Let $\pi = \pi_1(X)$ be the fundamental group of a connected CW complex X and let ρ be a representation of π to G. Note that via ρ, A can be viewed as a π-module, and also that A acts on the set of lifts of ρ to E via conjugation. When cohomology groups have A coefficients twisted by the action of π, the coefficients will be denoted by $\{A\}$. The following is a simple result in group cohomology; for reference material, see [11].

Proposition 1. There is a 1-1 correspondence between A-conjugacy classes of lifts of ρ and the cohomology group $H^1(\pi, \{A\})$.

Proof. The argument consists of relating the classification of liftings to that of splittings of a related extension, and then calling on standard cohomological results on the classification of splittings.

There is a pullback of the extension E to an extension $E^* = \rho^*(E)$ of π by A. The explicit definition of E^* is:

$$E^* = \{(x, e) \in \pi \times E | \rho(x) = p(e)\}$$

That E^* maps onto π via the projection on the first coordinate, and that the kernel is isomorphic to A, are both immediate, as are the following observations: (1) The extension $E^* \longrightarrow \pi$ is split by the map $s^* : x \longrightarrow (x, s(\rho(x)))$. (2) The actions of π on A via the splitting s^* and via the map $s \circ \rho$ are the same. (3) Any splitting s^* of $E^* \longrightarrow \pi$ determines a lifting of ρ via the map of E^* to E given by projection on the second factor. (4) Conversely, liftings $\tilde{\rho}$ of ρ determine splittings of $E^* \longrightarrow \pi$, given by the map $x \longrightarrow (x, \tilde{\rho}(x))$. (5) This correspondence induces a correspondence between A-conjugacy classes of liftings of ρ and A-conjugacy classes of splittings of $E^* \longrightarrow \pi$. Proposition 1 now follows from the classification of A-conjugacy classes of splittings by $H^1(\pi, \{A\})$. □

A consequence that yields the connection between the lifting problem and the structure of X is the following:

Proposition 2. $H^1(\pi, \{A\}) = H^1(X, \{A\})$.

Proof. Since X is a complex with fundamental group π, the classifying space for π is constructed from X by adding cells of dimension 3 and higher. Adding these cells leaves H^1 unaffected. □

3. Calculation of $H^1(X, \{A\})$ in Terms of Covering Spaces

In the present setting, the most important G-modules A that arise are related to the following construction. Specific examples will appear in the next section. Let H be a finite index subgroup of G. Let R be a commutative ring with unity; Z_n will suffice for all applications here. There is an associated G-module, denoted here by $R[G/H]$, defined as the set of formal finite sums $\Sigma r_i s_i$, where the s_i are left cosets of H. G acts naturally by left multiplication. (For an alternative description of such modules, suppose that G acts transitively on a set S. Then one can construct the G-module R^S in the natural way. This module is isomorphic to $R[G/H]$, where H is the stabilizer in G of some fixed element in S.)

Theorem 3. $H^1(X, \{R[G/H]\})$ is isomorphic to the untwisted cohomology $H^1(X_H, R)$, where X_H is the (not necessarily connected) cover of X associated to H.

Proof. Suppose first that ρ is onto, so that X_H is the connected cover of X associated to the subgroup $\rho^{-1}(H)$. Since ρ is onto, $R[G/H]$ is isomorphic to $R[\pi/\rho^{-1}(H)]$. Hence, the claim is that $H^1(X, \{R[\pi/\rho^{-1}(H)]\})$ is isomorphic to $H^1(X_H, R)$, or, in terms of group cohomology, $H^1(\pi, \{R[\pi/\rho^{-1}(H)]\})$ is isomorphic to $H^1(\rho^{-1}(H), R)$. In this form the result is basically Shapiro's Lemma. A proof with $R = Z$ is given in [11, Chapter 3, Section 6]. The proof for general R is the same. □

If ρ is not onto, the covering space theory is more complicated; see [12] for general reference. We first recall that X_H is defined to be the pullback to X of the universal covering of classifying spaces $K(H,1) \to K(G,1)$. In general X_H need not be connected; in fact, its components correspond to orbits of H in the coset space G/H under the action of π via ρ . The components and correspondence can be described explicitly as follows. The orbit of a coset of H, say gH, corresponds to a component of X_H, say X_{gH}, where X_{gH} is the connected cover of X corresponding to the subgroup $\rho^{-1}(gHg^{-1})$ contained in π. Hence, $H^1(X_H, R)$ splits as a direct sum corresponding to these components.

On the other hand, $R[G/H]$ no longer need be isomorphic to $R[\pi/\rho^{-1}(H)]$ as a π-module. In fact, $R[G/H]$ splits as a direct sum of π-modules corresponding to the orbits of the coset H in G/H under the action of π via ρ . The orbit of a coset of H, say gH, forms a π-module isomorphic to $R[\pi/gHg^{-1}]$. Shapiro's Lemma now applies to each of these summands and the corresponding components of the cover X_H to yield the desired result.

$R[G/H]$ contains a norm element Σs_i, the sum of all cosets. This norm element generates an invariant G-submodule, N, of $R[G/H]$. The quotient $R[G/H]/N$ will soon be seen to be of particular interest.

Theorem 4. If $H^2(X,R) = 0$ then $H^1(X, \{R[G/H]/N\})$ is isomorphic to $H^1(X_H, R)/p^*(H^1(X,R))$, where p is the covering map of X_H onto X.

Proof. There is an exact sequence of π-modules,

$$0 \to N \to R[G/H] \to R[G/H]/N \to 0,$$

and N is isomorphic to the trivial π-module R. The result follows from the associated long exact sequence on cohomology,

$$H^1(X,N) \to H^1(X, \{R[G/H]\}) \to H^1(X, \{R[G/H]/N\}) \to 0.$$

Theorem 3 identifies $H^1(X, \{R[G/H]\})$ with $H^1(X_H, R)$. That the first map corresponds to p^* depends on an analysis of the proof of Shapiro's Lemma. □

4. Basic Applications

This section presents simple applications of Theorems 3 and 4 to the situation of classical knots; that is, $\pi = \pi_1(S^3 - K)$ for some knot K. In each case E will denote the extension of G by the G-module $R[G/H]/N$, where N is the norm element defined above.

(i) $G = Z_2$, $R = Z_q$, and $H = 0$:

In this case $E = D_{2q}$, the dihedral group. By Theorem 3 and Theorem 4, lifts of the canonical map of π onto Z_2 to maps of π to D_{2q} correspond to $H^1(X_2, Z_q)/p^*(H^1(S^3 - K, Z_q))$, where X_2 denotes the 2-fold cyclic cover of the knot complement. In the case that q is odd, this quotient of cohomology groups is easily seen to give the first cohomology of the 2-fold branched cover of S^3 branched over K. This recovers the classical relationship between dihedral covers and the first homology of the 2-fold branched cover.

(ii) $G = Z_3,\ R = Z_2$, and $H = 0$:

In this case, $E = A_4$. As in case 1, we now see that lifts of the canonical representation of π onto Z_3 correspond to $H^1(\overline{X}_3, Z_2)$, where $\overline{X}_3$ is the 3-fold cyclic branched cover of S^3 branched over K. (Lifts that correspond to nontrivial cohomology classes are automatically onto A_4 so we recover a result of Riley [8] that a knot group maps onto A_4 if and only if $H_1(\overline{X}_3)$ contains 2-torsion. Riley expressed this condition in terms of the Alexander polynomial.)

(iii) $G = S_3,\ R = Z_2$, and $H = \{1, (12)\}$:

In this case one computes that $E = S_4$. The cover of interest is the 3-fold irregular cover of $S^3 - K$. This space has two torus boundary components, so the Z_2-cohomology of the cover has rank at least 2. Since the rank of the cohomology of $S^3 - K$ is 1, the quotient $H^1(X_H, R)/p^*(H^1(X, R))$ has rank at least 1, and in particular is nontrivial. Hence, if a knot group represents onto S_3 that representation lifts nontrivially to a representation to S_4. It is readily checked that any such lift is surjective. Perko's result [1] that if a knot group maps onto S_3 then it maps onto S_4 is recovered. Replacing G with D_{2p}, H with the index p subgroup, and R with Z_n, yields a generalization of Perko's result that was extensively examined by Hartley [3], where the detailed structure of these groups was also examined.

(iv) $G = Z_p,\ R = Z_q$, and $H = 0$, with p and q relatively prime:

Here E is a metabelian group, and the liftings correspond to cohomology classes of the p-fold cyclic branched cover of S^3 branched over K, recovering other results of Hartley [2]. Note that this does not represent the most general metabelian group, for instance, the module $R[G/H]/N$ may contain G-invariant submodules, in which case a more detailed analysis of the Z_q homology of the p-fold cover (as a Z_p-module) is called for; see [2] for a detailed analysis. The case that p and q are distinct primes illustrates the possibilities, and also provides information on metacyclic representations, first investigated by Fox [4]. (See also [7], and [13] for a general discussion of metabelian representations.)

First suppose that p divides $q - 1$. Then the field Z_q contains $p - 1$ distinct nontrivial p-roots of unity. Let $t \in E$ represent a generator of the image of Z_p under the splitting. Multiplication by t defines an automorphism of E, a Z_q vector space. It now readily follows that E decomposes as the direct sum of the $(p-1)$ eigenspaces of the action. This decomposes E as the direct sum of $(p - 1)$ semidirect products of Z_q and Z_p, the nontrivial metacylic groups. Finally, it can be shown that the liftings to each of these metacyclic extensions correspond to the eigenspaces of the (Z_p) covering translations acting on the (Z_q) homology of the p-fold cyclic branched cover of S^3 branched over K.

On the other hand, if p does not divide $q-1$, the situation is more subtle. If q has order f mod p then it can be shown that E splits as the direct sum of $(p-1)/f$ metabelian extensions of Z_p. Again in this case, the homology of the branched cover yields information on the liftings to these metabelian groups.

5. Peripheral Subgroups

In studying the representations of knot groups it is natural to investigate the value of the representation on the meridian of the knot, or, more generally, the restriction of the representation to the peripheral torus in the knot complement. In the case that the representation under consideration arises as the lift of another representation, the interpretation of the restriction of the lift to the meridian or peripheral torus in terms of the obstruction theory is immediate; this restriction corresponds to the restriction map on cohomology, from $H^1(X_H, R)/p^*(H^1(X, R))$ to $H^1(\tilde{Y}, R)/p^*(H^1(Y, R))$, where X is the knot complement, Y is the meridian or peripheral torus of the knot, and $\tilde{Y}$ is the preimage of Y in X_H. In this section we will analyze just one example of the methods involved here, the case of S_3 and S_4. More basic examples are easily analyzed without the aid of the obstruction theory interpretation, more complicated examples introduce technical details but no new insights.

Theorem 8 states that every such representation has a cyclic lift, one for which the meridian maps to a 4-cycle. This was conjectured by Perko [1] and proved by Rickard [9, unpublished] using far different techniques than those below. In Theorem 9 we show that the number of conjugacy classes of cyclic lifts equals the number of conjugacy classes of simple lifts, those for which the meridian maps to a transposition. This result was stated by Thistlethwaite [10, page 68], in somewhat different form; Thistlethwaite states that the number of surjective S_3 representations plus the number of surjective simple S_4 representations equals the number of surjective cyclic S_4 representations. The equivalence of the two statements follows easily, noting that every simple lift to S_4 of a representation onto S_3 is either onto, or has image isomorphic to S_3. (The proof of Theorem 9 given here is far different from Thistlethwaite's approach.)

Before working with the knot complement and the peripheral torus, consider the case of $X = S^1$, and suppose that π maps the generator of $\pi_1(X)$ to either the identity element, (1), or a transposition, say (12). Before stating the lemma, note that $Z_2[S_3/H]/N$ is isomorphic to $Z_2 \times Z_2$, with S_3 acting by permuting the three nontrivial elements.

Lemma 5. $H^1(S^1, \{Z_2 \times Z_2\})$ is isomorphic to $(Z_2)^2$ or (Z_2), depending on whether or not the representation is trivial or onto H. In the first case, the 3 nontrivial elements correspond to lifts of the representation that send the generator to products of two disjoint 2-cycles in S_4. In the second case, the nontrivial lift sends the generator to a 4-cycle.

Proof. These results can be obtained using covering spaces considerations, by basic results from cohomological group theory, or by analyzing the lifting problem in this simple setting by working explicitly with the groups involved. Details are left to the reader. □

For a knot group with representation onto S_3 there is a 3-fold irregular branched cover with two branch curves, one of index 1 and the other of index 2. The cover of the knot complement, $\tilde{X}_H$, hence has two boundary components, T_1 and T_2; both are covers of the peripheral torus. Observe that $H_1(T_1 \cup T_2, Z_2)$ is generated by 4 elements, m_1, m_2, l_1, and l_2, appropriate components of the preimages of the meridian and longitude. Working with the dual classes in cohomology we have 4 elements, μ_1, μ_2, λ_1, and λ_2. The image of the restriction map of $H^1(\tilde{X}_H, Z_2)$ to $H^1(T_1 \cup T_2, Z_2)$ includes μ_1, and μ_1 is in the image of p^*. (Consider the dual to the lift of the Seifert surface for K.)

A standard argument shows that the image of the restriction map must also contain a second element which cups against μ_1 trivially. Modulo μ_1 the only possibilities are μ_2, λ_1, or $\mu_2 + \lambda_1$. Soon we will show that the element λ_2 cannot be in the image. First we need the following result; the proof is a calculation that is left to the reader.

Theorem 6. Suppose that ρ is a representation of a knot group to S_3 with $\rho(m) = (12)$ and suppose that $\tilde{\rho}$ is a lift of ρ corresponding to an obstruction class $z \in H^1(X_H, Z_2)$. After modding out by the image of p^* the restriction of z to $H^1(T_1 \cup T_2, Z_2)$, say z', has the four possible values listed below, with the corresponding restrictions of $\tilde{\rho}$ to the peripheral subgroup as indicated.

(1) 0	$\tilde{\rho}(m) = (12)$	$\tilde{\rho}(l) = (1)$
(2) μ_2	$\tilde{\rho}(m) = (1324)$	$\tilde{\rho}(l) = (1)$
(3) λ_2	$\tilde{\rho}(m) = (12)$	$\rho(l) = (12)(34)$
(4) $\mu_2 + \lambda_2$	$\tilde{\rho}(m) = (1324)$	$\tilde{\rho}(l) = (12)(34)$

(Note that there are of course other possible values for the lifts, the others are conjugate to these.)

Lemma 7. Case 3 cannot occur.

Proof. In [14] it is proved that no representation of a knot group has these peripheral properties. □

Theorem 8. Every surjective representation of a knot group to S_3 has a cyclic lift.

Proof. By the discussion preceding the Theorem, one of case 2, 3, or 4 must occur for some lift. The last lemma rules out case 3, the other two cases correspond to cyclic lifts. □

Theorem 9. The number of conjugacy classes of cyclic lifts of the representation equals the number of conjugacy classes of simple lifts.

Proof. Let M denote the neighborhood of the meridian, and consider the exact sequence on cohomology:

$$H^1(X, M, \{Z_2 \times Z_2\}) \to H^1(X, \{Z_2 \times Z_2\}) \to H^1(M, \{Z_2 \times Z_2\})$$

The last group is isomorphic to Z_2, and the previous results show the second map is onto; the cyclic lift given by the previous theorem corresponds to a class that restricts nontrivially.

The kernel of the restriction map is hence of index two, and since these are Z_2 vector spaces, the order of the kernel is one half the order of $H^1(X, \{Z_2 \times Z_2\})$. That is, half the lifts restrict trivially (are simple) and half the lifts restrict nontrivially (are cyclic). □

An interesting corollary follows from the Theorem 6.

Corollary 10. Suppose that a knot group has a representation ρ onto S_3, then either every cyclic lift of ρ is trivial on the longitude or every cyclic lift of ρ is non-trivial on the longitude.

Proof. In Theorem 6, case 2 or case 4 occurs depending on whether μ_2 or $\mu_2 + \lambda_2$ is in the image of $H^1(X_H, Z_2)$ in $H^1(T_1 \cup T_2, Z_2)$. Both classes cannot be in the image, as μ_1 is in the image and the image is rank two. □

One obvious issue remains in the preceding analysis; which of the two possible cases, 2 or 4, occurs. It turns out that both can occur, as we now discuss.

Theorem 11. If the 3-fold irregular branch cover of S^3 associated to ρ is a Z_2-homology sphere, then every lift of ρ is nontrivial on the longitude to K. (That is, case 4 of Theorem 6 holds.)

Proof. The linking numbers of the branch curves in the cover equals 2(mod 4). (See for instance [15].) That is, an odd multiple (say r) of the index 2 branch curve bounds an oriented surface in the branched cover meeting the index 1 branch curve in s points, where $s \equiv 2(\text{mod } 4)$. This implies that $tm_2 + rl_2$ is homologous (via an oriented surface) to sm_1 in the irregular cover of the knot complement. Now consider the projection of that oriented surface to S^3. Since m_1 projects to m, m_2 projects to $2m$, and l_2 projects to l, we find that $(2t - s)m + rl$ is null homologous in the knot complement. Hence, $2t - s = 0$, and since $s \equiv 2(\text{mod } 4)$, t is odd.

Now working with Z_2 coefficients in the irregular cover, we see that $tm_2 + rl_2$ is null homologous. Since t and r are both odd, this is equivalent to $m_2 + l_2$ being null homologous (Z_2). Hence, case 4 applies as desired. □

It has been noted that the linking number of the branch curve in the irregular branched cover need not be 2(mod 4) if the cover is not a Z_2-homology sphere. The first example, the knot 10_{82}, was discovered by Perko [16]. One can check directly that this knot has a unique surjective S_4 representation (up to conjugacy). That representation is cyclic, and the longitude represents trivially. This example was discovered by a search; in another paper we will describe general techniques for the construction of further examples.

6. Other Applications

Apparently the methods presented apply more generally than to knots in S^3. Here we briefly comment on possible applications.

Other 3-manifolds For knots in homology spheres, the results of the previous section apply without modification. In the case that the 3-manifold is a rational homology sphere, M^3, and the order of the homology is prime to the order of E the results continue to hold without modification. However, if $H_1(M^3, Z)$ contains other torsion, some modifications are needed. For instance, a 3-fold irregular cover of the knot complement may now have a connected boundary, in which case the proof of Perko's Theorem, that representations of knot groups to S_3 lift to S_4, no longer holds.

Links The previous work carries over directly to apply to links in homology spheres. At only one point was the fact that we were working with a knot used; the hypothesis of Theorem 4, that $H^2(X, R) = 0$, may no longer be true. However, the map $H^2(X, N) \to H^2(X, \{R[G/H]\})$ is injective if N is a summand of $\{R[G/H]\}$, which is in fact the case in all the examples we have considered. Hence, the proof of Theorem 4 continues to apply.

Higher Dimensions The techniques continue to apply here. As one illustration, we consider Perko's theorem for the class of fibered knots in S^4. This class includes twist spun knots as a special case; see [17] for a discussion of of twist spun knots and fiberings.

Suppose that K is a fibered knot in S^4, with fiber a 3-manifold, F^3. Let $\pi = \pi_1(S^4 - K)$ and suppose that there is a representation ρ of π to S_3. (Such a representation will exist if and only if the monodromy of the fibration, acting on $H_1(F^3, Z_3)$, has an eigenvalue of -1.) Then it can be shown that ρ maps $\pi_1(F^3)$ onto $Z_3 \subset S_3$, and the 3-fold irregular cover of $S^4 - K$ is fibered by the associated 3-fold cyclic cover of F^3. If that cover of F^3 has trivial first homology with Z_2 coefficients, then the first homology of the irregular cover of $S^4 - K$ is Z_2 (using Z_2 coefficients), and the map p^* is an isomorphism. Hence, by our previous results, ρ will not lift to a representation onto S_4.

The simplest example of this is the 2-twist spin of the trefoil knot. It is fibered by the punctured lens space, $L(3, 1)$, with 3-fold cyclic cover S^3. Hence its group admits a representation onto S_3 but not onto S_4. Of course, this particular example can be analyzed explicitly; its group is $< x, t | x^3 = 1, txt^{-1} = x^{-1} >$, a group that maps onto S_3 but not S_4.

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